Investigating Mathematical Ways of Reasoning and Knowing with Paperfolding

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I will describe the progress made towards developing a course, *Explorations in Geometry,* whose goals are to provide experience of mathematics as a distinctive way of knowing, perspectives on mathematical reasoning, and comparison of ways to reason and know in other disciplines. The course is "below" calculus and is intended for people whose curiosity about mathematics can be aroused, but aren't interested in the calculus sequence.

An element in *Explorations* that is key to accomplishing the course objectives, as well as being very popular, is a sequence of paperfolding projects. I begin by asking the students to do things with paper folding: fold a square from a scrap of paper, fold an equilateral triangle from a square, 'prove' the Pythagorean Theorem, fold a perpendicular bisector, fold a line parallel to an existing line, and also to fold some unit origami objects, such as the ones shown in Figure 1 below.

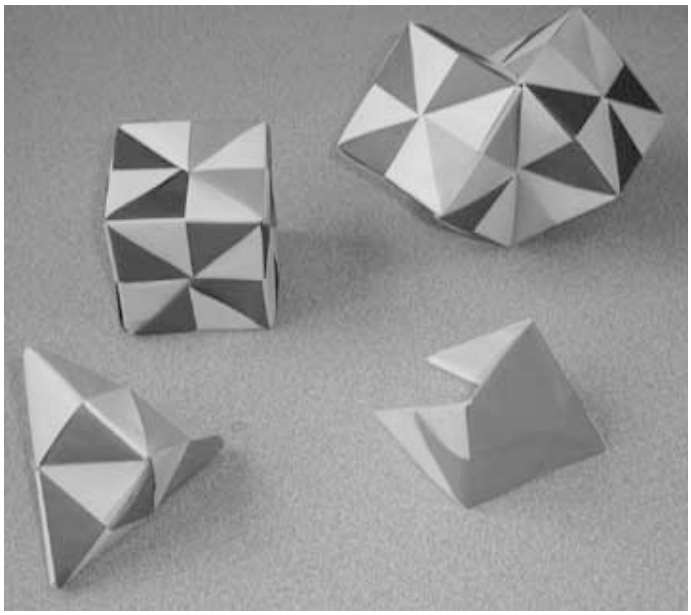

Figure 1

Folding Moves. After making quite a few constructions and origami objects with paper folding techniques, I ask the students to develop some ways to verbally describe, say to a ten-year-old, the basic 'folding moves' that one needs to know in order to do all the things that we have done as a class. The idea is to give students the experience of communicating mathematics. But this idea surprisingly leads to an illumination of the nature of axiom systems and a way to compare axiom systems for different geometries. The folding directions eventually become part of an axiom system for a *folding geometry*.

Through folding, I enable students to have discussions about such concepts as mathematical systems, undefined terms, axioms, and theorems, and consistency in a system of axioms. Students then compare mathematical systems to methods and standards of argument in the natural and social sciences, arts, and humanities.